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CURRENT DISTRIBUTION IN AN ARRAY OF SUPERCONDUCTING THIN FILMS

by I. D. Skurnick, J. H. Simmons, and A. R. Sass

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by I. D. Skurnick, J. H. Simmons, and A. R. Sass†

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SUMMARY

The conservation-of-fluxoid approach is used to derive the integral relations describing the current distribution in a general array of cylindrical London superconductors. This approach highlights properties of the kernel of the integral equation that permit the development of analytic solutions for a 2 by 2 array of closely spaced thin films of rectangular cross section. It was found that the current is distributed across the width of the film in a nonlinear, and nonsymmetric fashion, peaking at both edges. This peaking, in turn, dictates the switching characteristics of the system. It is shown that the dependence of the peaking on the separation distance between coplanar films can cause considerable variations in the critical current of the system I_c as this separation is varied from 1 to 20 times the film thickness.

INTRODUCTION

A major problem in the design of thin film superconductive devices has been the difficulty in predicting the maximum current that can be carried by the system before it switches to the normal state. Studies of the critical current in single superconducting thin films revealed that the problem is complicated by a nonlinear peaking of the current density at the film edges (refs. 1, 2, and 3). Recently Sass and Skurnick, employing the concept of fluxoid conservation, derived the inhomogeneous Fredholm integral equation of the second kind, which describes the current density in both single and double cylindrical superconductors (ref. 4). The properties of the Fredholm kernel, in turn, permitted them to obtain a closed-form solution to the double-film case, a superconducting thin-film strip transmission line. In the strip line, as in the single film, the current density varied in a distinctly nonlinear fashion near the edges of the system. With respect to its widthwise variation, the current distribution in both cases is symmetric with respect to the center of each film. In this report the fluxoid conservation approach is used to extend the integral relations for the current distribution to a general array of cylindrical superconductors.

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†R.C.A. Laboratories, Princeton, New Jersey.

The behavior of the Fredholm kernel permits the derivation of closed-form solutions for many cases of interest, in particular, the instructive example of a 2 by 2 matrix of thin films. The solution indicates, significantly, that the current distribution is strongly dependent on the proximity of neighboring films provided that such separations are on the order of 20 film thicknesses or less. This, in turn, suggests that the results obtained can be applied with reasonable accuracy to the more general case of a 2 by n matrix of thin films.

SYMBOLS

[The rationalized meter-kilogram-second system of units is used herein.]

$\hat{a}$	unit vector (treated as dimensionless quantity)
C	constant determined by total current
d	thickness of film
$\vec{E}$	electric field intensity
$\vec{H}$	magnetic field intensity
I	total current in individual film
$\vec{J}$	current per unit area in x-y plane
K(u,u')	kernel of one-dimensional Fredholm integral equation
k	l/d
l	distance between centers of two films in strip-line pair
m	distance between centers of coplanar films
P	$\ln\|(x - x')^2 + (y - y')^2\|$
q - 1	separation parameter, $q = m/w$
$\vec{r}$	field point in x-y plane
$\vec{r}'$	source point in x-y plane
u	dimensionless field point in one-dimensional analysis, $(2/w)y$
u'	dimensionless source point in one-dimensional analysis, $(2/w)y'$
w	width of film
x,y	Cartesian coordinates equivalent to $\vec{r}$, when primed equivalent to $\vec{r}'$

β^{-1} London penetration depth

$\kappa(u)$ coupling factor, $\int_{-1}^{1} K(u,u')du'$

λ $(\beta d)^2(w/d)(1/8\pi)$

μ_o permeability of free space, $4\pi\times10^{-7}$ H/m

Subscripts:

a array a

b array b

z z-component of vector

GENERAL ARRAY

Consider the array of parallel cylindrical superconductors shown in figure 1. In the following analysis the cross section of each superconductor is

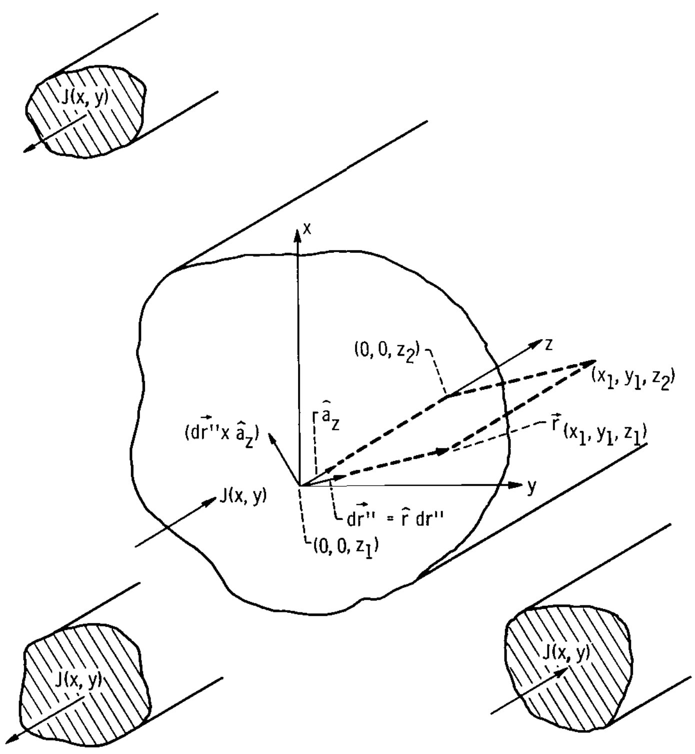


Figure 1. - General array of infinitely long cylindrical superconductors. Direction of current flow in cylinders is arbitrary. (One cylinder has been enlarged to show typical contour of integration.)

assumed arbitrary. If a direct current or low-frequency alternating current is pumped through the conductors, a quasi-static analysis is valid. The current is assumed to be in the z-direction; thus

$$\vec{J} = \hat{a}_z J_z(x,y)$$

Since the London equation $\vec{E} = \mu_o \beta^{-2}(\partial\vec{J}/\partial t)$, which relates the electric field $\vec{E}$, the current density $\vec{J}$, and the London penetration depth β^{-1}, is valid in each of the superconducting cylinders, application of Faraday's induction law to the dotted contour shown in figure 1 yields

$$\frac{d}{dt}\left[J(\vec{r}) - J(0) - \beta^2 \int_0^{\vec{r}} \vec{H}(\vec{r}\,'') \cdot (\hat{a}_z \times d\vec{r}\,'')\right] = 0 \tag{1}$$

where $\hat{a}_z$ is the unit vector in the z-direction and $d\vec{r}\,''$ is a differential vector line element of integration. The quantity contained within the brackets in equation (1) is the London fluxoid; the vanishing of its time derivative expresses the principle of fluxoid conservation. Starting from zero field initial conditions requires the fluxoid associated with any contour contained within the superconductor to vanish. Thus,

$$J(\vec{r}) = J(0) + \beta^2 \int_0^{\vec{r}} \vec{H}(\vec{r}\,'') \cdot (\hat{a}_z \times d\vec{r}\,'') \tag{2}$$

The magnetic field at a point $\vec{r}\,''$ is readily determined by summing over the contributions made by the current elements in all the cylinders. Hence,

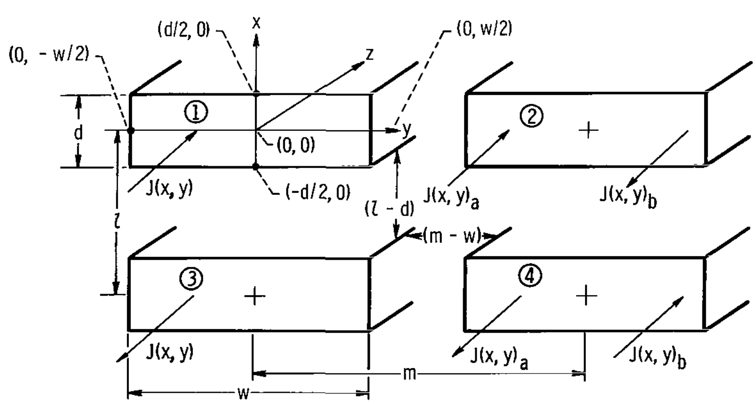


Figure 2. - Four superconducting thin films with current directions indicated for arrays a and b. In array a the current enters system through films 1 and 2 and leaves through films 3 and 4; in array b the current enters through films 1 and 4 and leaves through films 2 and 3.

$$\vec{H}(\vec{r}'') = \iint\limits_{\text{(all space)}} J(\vec{r}') \frac{[\hat{a}_z \times (\vec{r}'' - \vec{r}')]}{2\pi|\vec{r}'' - \vec{r}'|^2} d^2\vec{r}' \tag{3}$$

Substituting equation (3) into equation (2) leads to

$$J(\vec{r}) = C + \frac{\beta^2}{2\pi} \iint\limits_{\text{(all space)}} J(\vec{r}')\ln|\vec{r} - \vec{r}'|d^2\vec{r}' \tag{4}$$

where C is a constant determined by the total current.

From equation (2) it is clear that the nonuniformity of the current density in the cylinders is due to the net magnetic flux per unit length through areas bounded by contours of the type described in figure 1. It should be expected then that if this flux is small, the current density would exhibit only mild variations in the cylinders.

FOUR-FILM ARRAY

The particular array shown in figure 2 depicts a 2 by 2 matrix of superconducting thin films of rectangular cross section. The array can also be regarded as either two strip transmission lines in close proximity or two coplanar thin films deposited above a superconducting ground plane. Each film has a thickness d and width w. The spacing between the coplanar films is $m - w$, and the distance between the centers of a film and its image in a strip-line pair is l. The analysis is made for film parameters in a range of practical interest; that is, $d/w \leq 0.01$ and $\beta d \leq 1$.

Applying equation (4) to the four film array yields

$$\begin{aligned} J(x,y) = C + \frac{\beta^2}{4\pi}\Bigg[&\int_{-w/2}^{w/2}\int_{-d/2}^{d/2} J_1(x',y')P\,dx'\,dy' \\ &+\int_{m-(w/2)}^{m+(w/2)}\int_{-d/2}^{d/2} J_2(x',y')P\,dx'\,dy' \\ &+\int_{-w/2}^{w/2}\int_{-(d/2)-l}^{(d/2)-l} J_3(x',y')P\,dx'\,dy' \\ &+\int_{m-(w/2)}^{m+(w/2)}\int_{-(d/2)-l}^{(d/2)-l} J_4(x',y')P\,dx'\,dy'\Bigg] \end{aligned} \tag{5}$$

where $P = \ln|(x - x')^2 + (y - y')^2|$. Figure 2 shows that the arrays a and b possess the following symmetry properties:

$$\left.\begin{array}{lll} \text{Array a} & J(x,y) = -J(-x - l,y) = J(x,m - y) = -J(-x - l,m - y) \\ \text{b} & J(x,y) = -J(-x - l,y) = -J(x,m - y) = J(-x - l,m - y) \end{array}\right\} \tag{6}$$

Equation (5) can thus be greatly simplified by the following changes of variables:

$$\left.\begin{array}{ll} \text{Film } 2 & y' \to m - y' \\ 3 & x' \to -(x' + l) \\ 4 & \text{both } y' \to m - y' \text{ and } x' \to -(x' + l) \end{array}\right\} \tag{7}$$

Incorporating expressions (6) and (7) into equation (5) leads to a more compact description of the current density in the films:

$$J_{a,b}(x,y) = C + \frac{\beta^2}{4\pi} \int_{-w/2}^{w/2} \int_{-d/2}^{d/2} J(x',y') \times \ln\left\{\left[\frac{(x - x')^2 + (y - y')^2}{(x + x' + l)^2 + (y - y')^2}\right] F_{a,b}\right\} dx'\, dy' \tag{8a}$$

where

$$F_a = \frac{(x - x')^2 + (y + y' - m)^2}{(x + x' + l)^2 + (y + y' - m)^2} \quad \text{and} \quad F_b = \frac{1}{F_a} \tag{8b}$$

Since $d \leq \beta^{-1}$, it will be assumed for simplicity that J does not vary significantly with x. This assumption is not crucial and will be discussed in more detail at the end of this section. There is thus no loss of generality in setting $x = 0$. Integrating with respect to x', letting $\lambda = (\beta d)^2(w/d)(1/8\pi)$, $k = l/d$, $q = m/w$, and changing variables such that $u = 2y/w$ yield

$$J_{a,b}(u) = C + \lambda \int_{-1}^{1} J(u')K_{a,b}(u,u')du' \tag{9a}$$

where

$$
\begin{aligned}
K_{a,b}(u,u') = {} & \left\{ \ln\left[\left(\frac{d}{w}\right)^2 + (u - u')^2\right] + \left(k - \frac{1}{2}\right)\ln\left[(2k - 1)^2\left(\frac{d}{w}\right)^2 + (u - u')^2\right] \right. \\
& \left. - \left(k + \frac{1}{2}\right)\ln\left[(2k + 1)^2\left(\frac{d}{w}\right)^2 + (u - u')^2\right]\right\} + \left\{\frac{u - u'}{\frac{d}{w}}\left[2 \tan^{-1} \frac{\frac{d}{w}}{u - u'} \right.\right. \\
& \left.\left. + \tan^{-1} \frac{(2k - 1)\frac{d}{w}}{u - u'} - \tan^{-1} \frac{(2k + 1)\frac{d}{w}}{u - u'}\right]\right\} \pm \left\{\ln\left[\left(\frac{d}{w}\right)^2 + (u + u' - 2q)^2\right] \right. \\
& + \left(k - \frac{1}{2}\right)\ln\left[(2k - 1)^2\left(\frac{d}{w}\right)^2 + (u + u' - 2q)^2\right] - \left(k + \frac{1}{2}\right)\ln\left[(2k + 1)^2\left(\frac{d}{w}\right)^2 \right. \\
& \left.\left. + (u + u' - 2q)^2\right]\right\} \pm \left\{\left(\frac{u + u' - 2q}{\frac{d}{w}}\right)\left[2 \tan^{-1}\left(\frac{\frac{d}{w}}{u + u' - 2q}\right) \right.\right. \\
& \left.\left. + \tan^{-1} \frac{(2k - 1)\frac{d}{w}}{u + u' - 2q} - \tan^{-1} \frac{(2k + 1)\frac{d}{w}}{u + u' - 2q}\right]\right\}
\end{aligned}
\tag{9b}
$$

where the plus sign corresponds to array a and the minus sign to array b.

Clearly, current elements in all four films contribute to the total magnetic flux through any individual film. Figure 3 shows, however, that contributions from antisymmetric current elements in thin films tend to cancel them-

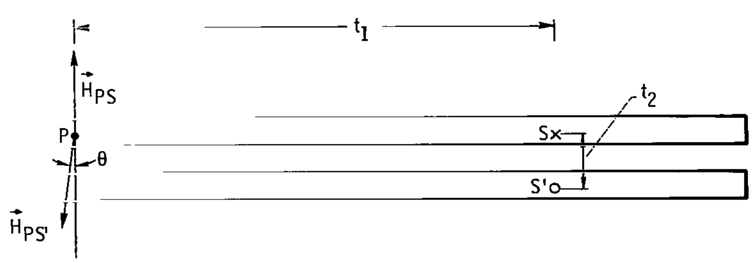
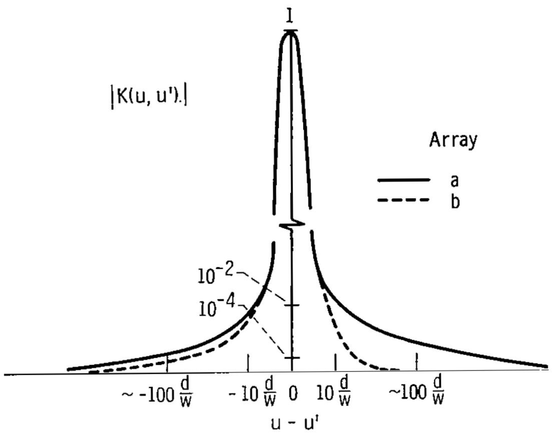
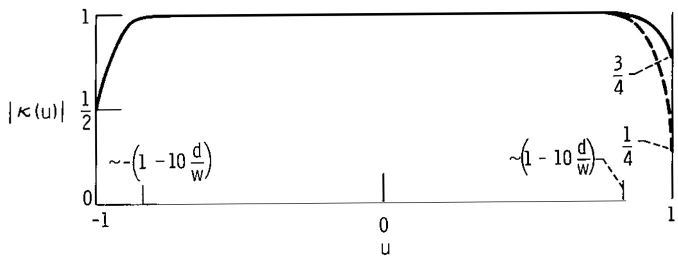


Figure 3. - Contributions to magnetic field at P from distant antisymmetric source elements. (Film widths are not drawn to scale.)



(a) Absolute value of kernel plotted for arrays a and b as function of u - u' for d/w = 0.001, k = 2.00, and q = 1.001. (Peak value has been normalized to unity.)



(b) Absolute value of coupling factor plotted against u for arrays a and b. (Parameters of system are the same as those used in fig. 4(a).)

Figure 4. - Kernel and coupling factor. (Abscissa not drawn to scale.)

selves out, particularly if these elements are distant from the field point under consideration. Let the currents at points S and S' be equal in magnitude but opposite in direction. If $\vec{H}_{PS}$ is the magnetic field at P due to the current at S, $\vec{H}_{PS'}$ the field at P due to the current at S', and $t_1 \geq 7t_2$, it follows that

$$\frac{|\vec{H}_{PS}| - |\vec{H}_{PS'}|}{|\vec{H}_{PS}|} \leq 0.010$$

and $0.989 \leq \cos\theta < 1.000$. Thus the great bulk of the flux through areas bounded by contours of the type shown in figure 1 is contributed by current elements comparatively close to the field point. It should be expected then that the kernel would be sharply peaked about $u = u'$ and decrease with increasing $|u - u'|$. Figure 4(a) shows representative one-dimensional kernels plotted as a function of $u - u'$. Although the kernels are not drawn to scale, the general features are evident and are supported by numerical analysis.

Since $l > d$ and $m > w$ it can be shown that the one-dimensional kernels in equation (9b) are negative definite. Thus, equation (9a) can be rewritten as

$$J(u) = C - \lambda \int_{-1}^{1} J(u')|K(u,u')|du' \tag{10}$$

Equation (10) can be solved by the method of successive approximations. Starting with $J_0(u) = C$ it can easily be shown that

$$J_n(u) = C\left[1 + \sum_{m=1}^{n} (-\lambda)^m \int_{-1}^{1}\int_{-1}^{1} \cdots \int_{-1}^{1} |K(u_{m-1},u_m)| \cdots |K(u,u_1)|du_m \cdots du_1\right] \tag{11a}$$

The current density is given by

$$J(u) = \lim_{n\to\infty} J_n(u) \tag{11b}$$

provided that the series in equation (11a) converges as $n \to \infty$. Thus, to the zeroth order in λ the current density is assumed to be constant in the films. The first order correction considers the interactions of all the source points in the films with a particular field point. The second order correction considers the effect of all the source points on an undisturbed source point, before the effect on a field point is computed. The higher order terms (in λ) are further corrections for the interaction between current elements.

It is helpful to evaluate the coupling factor defined by

$$\int_{-1}^{1} K(u,u_1)du_1 \equiv \kappa(u) \tag{12a}$$

From equations (9b) and (12a) the coupling factor can be written as

$$\kappa_{a,b}(u) = (A_1 \pm A_1')\Big|_{Q=1} + \left(k - \frac{1}{2}\right)(A_1 \pm A_1')\Big|_{Q=2k-1} - \left(k + \frac{1}{2}\right)(A_1 \pm A_1')\Big|_{Q=2k+1}$$

$$+ 2(A_2 \pm A_2')\Big|_{Q=1} + (A_2 \pm A_2')\Big|_{Q=2k-1} - (A_2 \pm A_2')\Big|_{Q=2k+1} \tag{12b}$$

where

$$A_1 = \ln \frac{\left[\left(Q\,\frac{d}{w}\right)^2 + (u+1)^2\right]^{u+1}}{\left[\left(Q\,\frac{d}{w}\right)^2 + (u-1)^2\right]^{u-1}} - 4 + 2\left(Q\,\frac{d}{w}\right)\left(\tan^{-1}\frac{u+1}{Q\,\frac{d}{w}} - \tan^{-1}\frac{u-1}{Q\,\frac{d}{w}}\right) \tag{13a}$$

$$A_1' = \ln \frac{\left[\left(Q\,\frac{d}{w}\right)^2 + (u-2q+1)^2\right]^{u-2q+1}}{\left[\left(Q\,\frac{d}{w}\right)^2 + (u-2q-1)^2\right]^{u-2q-1}}$$

$$- 4 + 2\left(Q\,\frac{d}{w}\right)\left(\tan^{-1}\frac{u-2q+1}{Q\frac{d}{w}} - \tan^{-1}\frac{u-2q-1}{Q\,\frac{d}{w}}\right) \tag{13b}$$

$$A_2 = (1 + u^2)\frac{\pi}{2}\frac{w}{d} - \frac{Q^2}{2}\frac{d}{w}\left\{\left[1 + \left(\frac{1 + u}{Q\frac{d}{w}}\right)^2\right]\tan^{-1}\frac{1 + u}{Q\frac{d}{w}}\right.$$

$$\left. + \left[1 + \left(\frac{1 - u}{Q\frac{d}{w}}\right)^2\right]\tan^{-1}\left(\frac{1 - u}{Q\frac{d}{w}}\right) - \frac{2}{Q\frac{d}{w}}\right\} \tag{13c}$$

and

$$A_2' = (2q - u)\pi\frac{w}{d} - \frac{Q^2}{2}\frac{d}{w}\left\{\left[1 + \left(\frac{2q - u + 1}{Q\frac{d}{w}}\right)^2\right]\tan^{-1}\frac{2q - u + 1}{Q\frac{d}{w}}\right.$$

$$\left. - \left[1 + \left(\frac{2q - u - 1}{Q\frac{d}{w}}\right)^2\right]\tan^{-1}\left(\frac{2q - u - 1}{Q\frac{d}{w}}\right) - \frac{2}{Q\frac{d}{w}}\right\} \tag{13d}$$

The plus sign refers to $\kappa_a(u)$ and the minus sign refers to $\kappa_b(u)$.

Because of the behavior of the kernel, $\kappa(u)$ must also be negative definite. Since the kernel has a rather narrow effective width (fig. 4(a)), $\kappa(u)$ should remain approximately constant over the central portion of the films. On the other hand, when the distance to the edge of the film is on the order of the width of the kernel, $\kappa(u)$ is a strong function of u. This implies that for all practical purposes field points in the central region of the film view the film as being infinitely wide. Figure 4(b) is a sketch of the absolute value of the coupling factor for typical values of the parameters of the system. Since $\kappa(u) \simeq \kappa(0)$ over most of the width of the film, the following approximation suggests itself

$$\int_{-1}^{1} |\kappa(u_1)|\,|K(u,u_1)|du_1 \simeq |\kappa(0)|\,|\kappa(u)| \tag{14}$$

Thus in general

$$\int_{-1}^{1}\int_{-1}^{1}\cdots\int_{-1}^{1} |K(u_{m-1},u_m)|\cdots|K(u,u_1)|du_m\cdots du_1 \simeq |\kappa(0)|^{m-1}|\kappa(u)| \tag{15}$$

Combining equations (15) and (11b) yields

$$J(u) = C\left\{1 - \lambda|\kappa(u)|\left[\sum_{m=0}^{\infty}(-1)^m(\lambda|\kappa(0)|)^m\right]\right\} \tag{16}$$

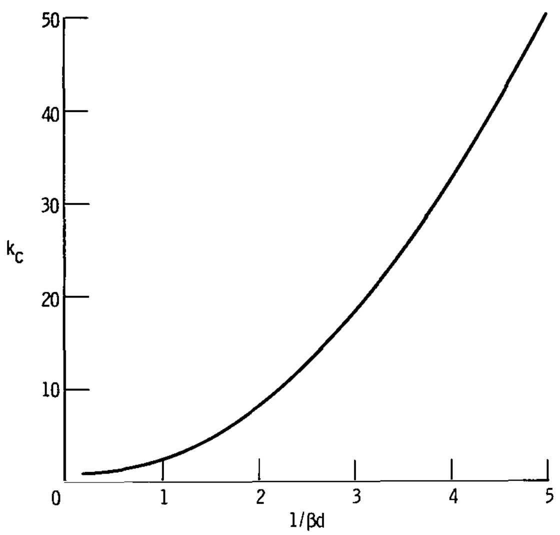


Figure 5. - Range of convergence for equation (16) for $d/w \leq 0.001$. In this limit k_c is independent of d/w. Permissible values of k are given by $k < k_c = [2/(\beta d)^2] + 0.25$.

If $\lambda|\kappa(0)| < 1$ the infinite series in equation (16) converges absolutely, and the current density can be described in closed form by

$$J(u) = C\left[1 - \frac{\lambda|\kappa(u)|}{1 + \lambda|\kappa(0)|}\right] \tag{17}$$

When $\lambda|\kappa(0)| \geq 1$, the series in equation (16) does not converge, hence the method of successive approximations is not satisfactory. Figure 5 illustrates the range of convergence for equation (16) for $d/w \leq 0.001$.

A factor of particular interest and importance is the relative peaking of the current density at the edges of the film, that is, $J(\pm 1)/J(0)$. In contrast to the single film and strip line, it is clear that the distribution in arrays a and b cannot be symmetric. Considering the qualitative features of the current distribution in the strip superconductor indicates that if the current in films 1 and 2 are in the same direction the greatest peaking should be expected on the outside edges of the system. On the other hand, if the current in films 1 and 2 run opposite to each other, the greatest peaking should occur on the inside edges. Figure 6 is a representative sketch of $J(u)$ against u for both cases.

From equation (17) the relative peaking at the edges is given by

$$J(\pm 1)/J(0) = 1 + \lambda\left[|\kappa(0)| - |\kappa(\pm 1)|\right] \tag{18}$$

In the limit where $d/w \leq 0.001$ it is possible to express the peaking in a form that explicitly demonstrates its dependence on various parameters of the system. From equations (12b), (13), and (18) the peaking on the outside edges can be shown to be

$$\frac{J_{a,b}(-1)}{J_{a,b}(0)} \simeq 1 + \frac{1}{2}\lambda|\kappa(0)| \simeq 1 + \frac{(\beta d)^2}{4}\left(k - \frac{1}{4}\right) \tag{19}$$

On the inside edges of the system equation (18) can be approximated by

$$\frac{J_{a,b}(1)}{J_{a,b}(0)} \simeq 1 + \frac{(\beta d)^2}{4}\left(k - \frac{1}{4}\right) \mp \frac{(\beta d)^2 k^2 \frac{d}{w}}{4\pi}\left[\frac{1}{q(q - 1)}\right] + \ldots \tag{20}$$

where the minus sign is for array a and the plus sign is for array b. It is apparent that the dependence of the peaking on the spacing between coplanar films is only important when $q - 1$ is on the order of $20(d/w)$ or less. For

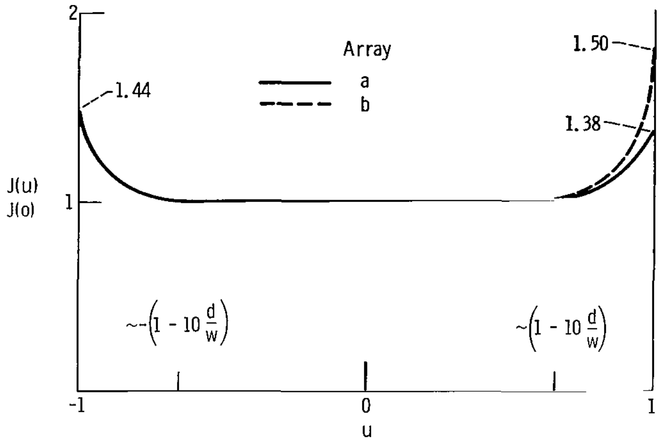


Figure 6. - Normalized current density plotted against u for d/w = 0.001, βd = 1, k = 2.00, and q = 1.005. (Edge regions of u-axis are not drawn to scale.)

all practical purposes, the peaking at the outside edges of the system is unaffected by the separation between the coplanar films. Equation (20) is an excellent approximation to equation (18) (i.e., it is in error by less than 1 percent) for $q - 1 \geq 5(d/w)$. For smaller separations higher order terms in d/w must be taken into consideration.

Since a superconductor starts to switch to the normal state when J exceeds a certain critical value, equations (19) and (20) and figure 6 indicate that the critical total current will show a stronger dependence on the separation parameter $q - 1$ in array b than in array a. In calculating the total current in the system the variation of J in the direction of the film thickness should be considered. If $I_{cu}^{(b)}$ is the critical total current in array b, calculated by assuming a uniform widthwise current density distribution in the films (no peaking at the edges), it is obvious that $I_c^{(b)} < I_{cu}^{(b)}$. From equation (20) it follows that

$$\frac{I_c^{(b)}}{I_{cu}^{(b)}} \simeq \left\{1 + \frac{(\beta d)^2}{4}\left(k - \frac{1}{4}\right) + \frac{(\beta d)^2 k^2 \frac{d}{w}}{4\pi}\left[\frac{1}{q(q - 1)}\right]\right\}^{-1} \tag{21}$$

Figure 7, plotted by combining equations (13) and (18), shows the dependence of $J_b(1)/J_b(0)$ on $m - w$ for a typical value of k. This figure is of particular importance because it also relates the critical current of the system (through the relative peaking) to the spacing between coplanar films. The general features of the curve are true for all values of d/w considered in this report. If $m - w > 20d$, $J_b(1)/J_b(0)$ and hence I_c remain approximately constant.

From equations (20) and (21) it is apparent that as $m - w$ is decreased $I_c^{(b)}$ can be greatly reduced, particularly if $k \simeq k_c$, the critical value for convergence of equation (16). Calculations show that if $\beta d = 1$ and $k = 2.00$, $I_c^{(b)}$ decreases by 15 percent as $m - w$ is decreased from 20d to d. If $\beta d = \frac{1}{2}$ and $k = 8.00$, $I_c^{(b)}$ decreases by 22 percent in this same interval. No dependence of $I_c^{(a)}$ on q was found, nor should it be expected, since the maximum peaking occurs on the outside edges of the system and consequently is insensitive to q. The variation of $I_c^{(b)}$ with $m - w$ can be reduced by selecting a smaller k (for fixed βd) or a smaller βd (for fixed k).

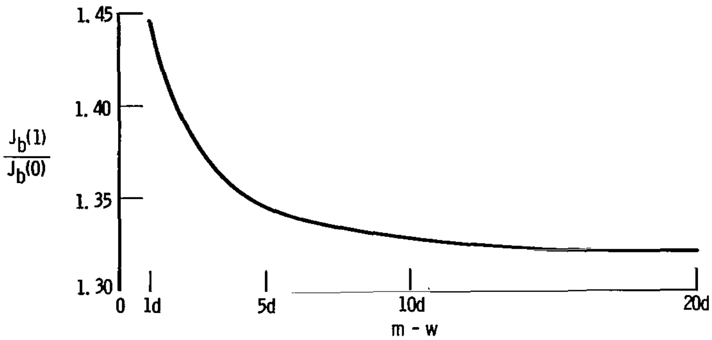


Figure 7. - Relative peaking of current density at inside edges of films in array b plotted against separation distance between adjacent films in units of film thickness for $\beta d = 1$, $k = 1.5$, and $d/w \leq 0.001$.

Two approximations were made in the development of the solution and require brief explanations. The first assumption was that J does not vary significantly with x. It is easily proven that in the central portion of the films $J \sim \cosh \beta(x - d/2)$, thus the assumption is not strictly correct. However, it is shown in reference 4 that if this variation is substituted into the original two-dimensional integral equation, the subsequent one-dimensional integral equation is of the same form as equation (9a), with λ replaced by $\alpha\lambda$. The parameter α is bounded, and depends on the thickness of the film in the following manner:

$$1 < \alpha < \frac{\sinh \frac{\beta d}{2}}{\frac{\beta d}{2}} \tag{22}$$

Since $\beta d \leq 1$, it follows that $1 < \alpha < 1.04$. This would result, at most then, in a 2 percent decrease for $J(u)$ in the central region of the films. This error is less near the edges of the films since in these regions $J(x)$ more nearly approaches the constant value implicit in the original assumption.

The second assumption was that equation (14) is valid in the edge regions of the films. Although figure 4(b) suggests that the approximation would not be accurate here, at the very least equation (17) would provide a reasonable trial value for any numerical solution of the integral equation. Starting from equation (17), numerical solutions obtained with the aid of an IBM 7094II computer revealed that the greatest error in the closed-form solution occurs when $k \simeq k_c$. In this region it was found that the use of equation (17) can result in an underestimate of I_c of as much as 10 percent. For $k \leq 0.65\ k_c$ this error is less than 5 percent. It should be noted that the percentage decrease in $I_c^{(b)}$, predicted by the closed-form solution, as $m - w$ is varied from 20d to d, agrees with that found by numerical analysis to better than 5 percent.

CONCLUDING REMARKS

It can be seen from equation (20) and figure 6 that the presence of a second strip line alters the current distribution in the original system only in what would be the inside edges of the new system. Consider now the more general case of a 2 by n matrix (i.e., n equally spaced films over a superconducting ground plane, or n closely spaced strip lines). If the direction of the current alternates from film to film in a given row, the critical current of any 2 by 2 submatrix should be the same as that found for an isolated 2 by 2 matrix. In this case the additional superconductors only serve to bring the current peaking on the outside edges of the submatrix up to the same value as appears on the inside edges. Since the maximum peaking occurred previously on the inside edge, the critical current of the submatrix would be unaltered. On the other hand, if the current is in the same direction in each of the coplanar films, the peaking on the outside edges would be decreased to the level found previously on the inside edges, thus increasing the critical current of the submatrix. With arguments such as these, equations (20) and (21) can be used to determine the critical current for the more general 2 by n array.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 6, 1965.

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